

SPHALERONS AND ELECTROWEAK STRINGS

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Abstract

We show that the sphaleron energy which identifies with the (instanton) potential barrier for B-violation is reduced in the presence (background) of an electroweak Z-string. We also show that for large enough Higgs coupling, λ , the sphaleron energy can go negative. For such λ , electroweak Z-strings can reduce their energy by accumulating sphaleron bound states. This further endows them with baryon number. Given our approximation, the value of λ at which this occurs is rather large to be realistic for the standard electroweak model.

INTRODUCTION

There has been a lively interest of late in several scenarios of baryon number (B) violation with an eye to the B-asymmetry of the universe. A particular saddle-point solution of the electroweak theory, the sphaleron [1–3], has been identified with the potential barrier for the instanton process that takes one from one pure gauge (PG) winding number vacuum to an adjacent one. Since the instanton process changes the winding number by one unit and the anomaly translates this into changes of B, these sphaleron solutions are relevant for B violating processes in the early universe.

More recently, vortex or string like solutions in the same electroweak model have been considered [4]. Further, it has been shown that a linked configuration of loops of such strings carries winding number (also termed Chern Simons number, N_{cs}) [5]. Such configurations can then be broken and twisted into two sphalerons which can decay to the vacuum of winding number, $N_{cs} = 0$, providing a B violating scenario. This could have relevance to the B asymmetry of the universe as these strings may have formed during the electroweak phase transition (EWPT) in the evolution of the universe.

Much earlier Nambu [6] introduced these electroweak strings in a different context, while considering a dumbbell shaped object in the electroweak theory that has a monopole and an antimonopole at the extremities joined by such a string or flux tube. Such an object could be stabilized by rotating it when the centrifugal force can balance the string tension which would normally contract the flux tube to zero. Nambu’s dumbbell is also one of the configurations encountered in the passage of twisted loop to two sphalerons [5].

There is a quite different development that we shift to now. It was pointed out by the author [7] that a sphaleron like configuration which has an asymptotic magnetic field

$$B^{sp}|_{\infty} = \frac{4}{e}\eta_{sp}\frac{\hat{e}_r \sin \theta}{r^2} \quad (1)$$

can interact with an extended magnetic field so as to bring down the energy of the sphaleron configuration very substantially. This happens via the long range attractive magnetic interaction that is controlled by the parameter, $B_0 R$, where B_0 is the uniform applied external

magnetic field and R is the typical length scale of the region over which the field is applied. When the control parameter $B_0 R \sim 10^8$ gauss cm, the attractive magnetic interaction can compete with the usual sphaleron mass and total energy of the sphaleron configuration comes down to zero or below. This observation has import for situations which carry extended magnetic fields, e.g. stars.

It seemed of interest to look upon the flux tubes of [6] or the EW strings of [4] and [5] in this light. However, there are some notable differences. The strings (flux tubes) of [4]- [6] carry SU(2) flux. Nambu considers strings with Z flux as they are energetically favored and [4,5] have strings with Z, W^+ , W_- flux, whereas above [7] we have considered real magnetic flux. Thus, there cannot be any long-range interactions with EW strings, since long range interactions are purely electromagnetic.

The range of interaction for the Z string is set by $1/m_Z \sim 10^{-16}$ cm. We expect this is the typical radius of the Z string. Given this radius and the usual flux quantum carried by the string we find an average Z magnetic field $B^Z \sim 10^{24}$ gauss for the Z string. The control parameter, $B^Z R \sim 10^8$ gauss cm is of the same order as that above, which brought sphaleron energies down to zero. Also, given that the entire Z magnetic field acts inside of a radius $1/m_Z$, within which the Z field is non-zero the lack of a long-range interaction may not scotch all optimism.

We will address the question of the interaction of a sphaleron with a Z string which may have been formed during the EWPT, to see if the sphaleron energy can be markedly lowered by its interaction with the string. The sphaleron energy is identified with the instanton potential barrier and if sufficiently lowered could catalyse B-violation. We find, though not for the expected range of values of the higgs mass, that the sphaleron energy can even go negative which implies that a sphaleron can bind to the string and lower its energy.

THE Z-STRING OR THE NAMBU STRING

The dumbbell object considered by Nambu [6] is made up of an SU(2) monopole which is separated from an SU(2) antimonopole with a flux tube running along the axis. In the limit that the separation becomes large we get a Z-string.

We adopt the convention in [6]. The relevant part of the electroweak lagrangian is

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^a G^{\mu\nu a} - \frac{1}{4}F_{\mu\nu}^0 F^{\mu\nu 0} - \frac{1}{2}(D_\mu\phi)^\dagger D_\mu\phi - \frac{1}{8}\lambda^2(\phi^\dagger\phi - F^2)^2 \quad (2)$$

where $m_W = gF/2$, $m_H = \lambda F$ and $F(\text{VEV}) = 250 \text{ GeV}$.

The Higgs configuration for the monopole is

$$\phi = F \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{i\phi} \end{pmatrix} \quad (3)$$

where θ is the polar angle and ϕ the azimuthal angle with the monopole at the origin.

The antimonopole is correspondingly

$$\bar{\phi} = F \begin{pmatrix} \sin(\theta/2) \\ \cos(\theta/2)e^{i\phi} \end{pmatrix} \quad (4)$$

Now, as observed by Nambu [6], in the electroweak theory, the monopole has a spreading SU(2) flux out of the pole given by, $-4\pi\eta/g$, and a semi-infinite flux tube attached to the pole which carries a SU(2) flux, $-4\pi\xi/g$, out of the pole, with the constraint

$$\eta + \xi = 1 \quad (5)$$

The U(1) field is sourceless and carries a spreading flux $4\pi\eta/g'$ and the same flux is returned via a semi-infinite flux tube.

When we connect the monopole with the antimonopole there is no flux at infinity. However, there is a flux tube connecting the two which carries an SU(2) flux $-4\pi\xi/g$ and a U(1) flux $-4\pi\eta/g'$. We shall go by the electromagnetic and Z field definitions given in [6] though they are not unique.

The EM and Z flux carried by the flux tube are then given by

$$\begin{aligned}\Phi_{\text{em}} &= -\sin \theta_W \Phi_{\text{SU}(2)} + \cos \theta_W \Phi_{\text{U}(1)} = \frac{4\pi}{e}(-\eta \cos^2 \theta_W + \xi \sin^2 \theta_W) \\ \Phi_Z &= -\cos \theta_W \Phi_{\text{SU}(2)} - \sin \theta_W \Phi_{\text{U}(1)} = \frac{4\pi}{e}(\sin \theta_W \cos \theta_W)\end{aligned}\quad (6)$$

where, as we have defined earlier,

$$\eta + \xi = 1, \quad e = g \sin \theta_W = g' \cos \theta_W \quad \text{and} \quad \tan \theta_W = g'/g.$$

Notice, that the Z flux, Φ_Z , above is quantized — it does not depend on η or ξ , whereas the EM flux, Φ_{em} , does. As Nambu observes a maximum reduction of flux field energy is gained if $\eta = \sin^2 \theta_W$ or $\xi = \cos^2 \theta_W$ leaving a purely Z flux tube connecting the pole and the antipole. This corresponds to a Z-string.

The energy per unit length of the flux tube is calculated by assuming that the Higgs field is zero inside the flux tube (of radius ρ) and the fluxes above are averaged over the cross-section to give a uniform field.

$$\frac{E}{L} = \int_0^{2\pi} \int_0^\rho d\phi d\rho' \left[\frac{\lambda^2}{8} (F^2)^2 + \frac{1}{2} B_{\text{SU}(2)}^2 + \frac{1}{2} B_{\text{U}(1)}^2 \right] \quad (7)$$

Now,

$$\begin{aligned}\vec{B}_{\text{SU}(2)} &= \Phi_{\text{SU}(2)}/\pi\rho^2 = -4\pi\xi/g\pi\rho^2 \cdot \hat{e}_Z \\ \vec{B}_{\text{U}(1)} &= \Phi_{\text{U}(1)}/\pi\rho^2 = -4\pi\eta/g'\pi\rho^2 \cdot \hat{e}_Z \\ \frac{E}{L} &= \pi\rho^2 \left[\frac{\lambda^2}{8} F^4 + 8 \left\{ \frac{\xi^2}{g^2} + \frac{\eta^2}{g'^2} \right\} \frac{1}{\rho^4} \right]\end{aligned}\quad (8)$$

Minimizing with respect to ρ^2

$$\begin{aligned}\pi \left[\frac{\lambda^2}{8} F^4 - \frac{8}{\rho^4} \left\{ \frac{\xi^2}{g^2} + \frac{\eta^2}{g'^2} \right\} \right] &= 0 \\ \rho_\lambda &= \left(\frac{8}{\lambda F^2} \right)^{1/2} \left[\frac{\xi^2}{g^2} + \frac{\eta^2}{g'^2} \right]^{\frac{1}{4}} \\ \frac{E}{L} &= 2\pi \left[\lambda F^2 \left\{ \frac{\xi^2}{g^2} + \frac{\eta^2}{g'^2} \right\}^{\frac{1}{2}} \right]\end{aligned}\quad (9)$$

One further minimizes with respect to the parameter ξ . This yields the same condition as above: $\xi = \cos^2 \theta_W$ or $\eta = \sin^2 \theta_W$; that is a purely Z flux tube or string. After minimization, the radius, $\rho_0(\lambda)$, and the energy per unit length, $\epsilon(\lambda)$, of the string are given by

$$\begin{aligned}\rho_0(\lambda) &= \left(\frac{8 \cos \theta_W \sin \theta_W}{\lambda F^2 e} \right)^{1/2} \\ \epsilon(\lambda) &= \frac{2\pi \lambda F^2 \cos \theta_W \sin \theta_W}{e}\end{aligned}\tag{10}$$

The Z magnetic field can be calculated from the flux on the assumption that it is uniformly distributed over the flux tube cross-section

$$|B_0^Z| = \frac{\Phi_Z}{\pi \rho_0^2(\lambda)} = \frac{4\pi \sin \theta_W \cos \theta_W}{e} \frac{\lambda F^2}{\pi \rho_0^2(\lambda)} = \frac{\lambda F^2}{2}\tag{11}$$

For convenience we can express these quantities in terms of their values for $\lambda = 1$ indicated by the subscript 1.

$$\begin{aligned}\rho_0(\lambda) &= \frac{1}{\sqrt{\lambda}} \rho_0(\lambda = 1) = \frac{1}{\sqrt{\lambda}} \rho_1 \\ \epsilon(\lambda) &= \lambda \epsilon(\lambda = 1) = \lambda \epsilon_1 \\ B_0^Z &= \lambda B_0^Z(\lambda = 1) = \lambda \frac{F^2}{2}\end{aligned}\tag{12}$$

THE INTERACTION OF THE SPHALERON CONFIGURATION WITH THE Z-STRING

The energy functional for the standard electroweak theory is

$$E = \int d^3x \left[\frac{1}{4} F_{ij}^0 F_{ij}^0 + \frac{1}{4} G_{ij}^a G_{ij}^a + \frac{1}{2} (D_i \phi)^\dagger (D_i \phi) + \frac{\lambda^2}{8} (\phi^\dagger \phi - F^2)^2 \right]\tag{13}$$

We shall work with the sphaleron configuration in [1,7] The asymptotic fields are given by

$$\begin{aligned}\phi|_\infty &= F \begin{pmatrix} \cos \theta \\ \sin \theta e^{i\phi} \end{pmatrix} \\ g\mathcal{A}_i^a|_\infty &= \frac{2(1-\eta_{sp})}{r} \sin \theta (\hat{e}_\parallel)^a (\hat{e}_\phi)_i - \frac{2}{r} \cos \theta (\hat{e}_\perp)^a (\hat{e}_\phi)_i - \frac{2}{r} (\hat{e}_\phi^a) (\hat{e}_\phi)_i \\ g'\mathcal{A}_i^0|_\infty &= \frac{2\eta_{sp}}{r} \sin \theta (\hat{e}_\phi)_i \\ (\hat{e}_\parallel) &= (\hat{e}_r) \cos \theta + (\hat{e}_\theta) \sin \theta \\ (\hat{e}_\perp) &= (\hat{e}_r) \sin \theta - (\hat{e}_\theta) \cos \theta\end{aligned}\tag{14}$$

First, we point out, that, above, the surviving asymptotic fields are electromagnetic and proportional to η_{sp} . The U(1) field is also explicitly proportional to η_{sp} . The η_{sp} dependent asymptotic magnetic field of the sphaleron configuration, which was crucial to provide the long range attractive interaction with the external magnetic field is no more relevant for its interaction with the Z-string as the latter does not carry the usual magnetic field. In fact, asymptotically, the η_{sp} dependent part does not contribute to the interaction of the sphaleron with the Z-string. We may therefore restrict ourselves to the pure SU(2) sphaleron, $\eta_{sp} = 0$, to make our estimates.

Note, that for the usual magnetic field case we have a sphaleron like configuration and not a solution as observed in [7]. In this case once $\eta_{sp} = 0$, we have the exact SU(2) sphaleron configuration, which is more respectable. The sphaleron asymptotic fields are,

$$\begin{aligned}\phi|_{\infty} &= F \begin{pmatrix} \cos \theta \\ \sin \theta e^{i\phi} \end{pmatrix} \\ g\mathcal{A}_i^a|_{\infty} &= \frac{2}{r} \sin \theta (\hat{e}_{\parallel})^a (\hat{e}_{\phi})_i - \frac{2}{r} \cos \theta (\hat{e}_{\perp})^a (\hat{e}_{\phi})_i - \frac{2}{r} (\hat{e}_{\phi}^a) (\hat{e}_{\phi})_i \\ g'\mathcal{A}_i^0|_{\infty} &= 0\end{aligned}\tag{15}$$

We use the ansatz [1–3] where the solution is obtained by multiplying the asymptotic fields, above, by appropriate radial functions which go to unity at ∞ and regulate the fields at the origin. These functions are $f(r)$ and $h(r)$ for the higgs and gauge fields respectively.

Now, instead of the usual magnetic field background, we need to have the Z magnetic field of the Z-string as the background. This is easily accomplished. The expressions for SU(2) and U(1) magnetic fields are

$$\begin{aligned}gB_i^a &= \frac{4(\hat{e}_r)_i}{r^2} [-h(r)(1-h(r))] [\cos \theta (\hat{e}_{\parallel})^a + \sin \theta (\hat{e}_{\perp})^a] \\ &\quad + \frac{2}{r} \frac{dh(r)}{dr} \left\{ [(\hat{e}_{\theta})_i [-\sin \theta (\hat{e}_{\parallel})^a + \sin \theta (\hat{e}_{\perp})^a]] - (\hat{e}_{\phi})_i (\hat{e}_{\phi})^a \right\} \\ &\quad - e|B_0^Z| (\hat{e}_Z)_i (\hat{e}_{\parallel})^a \cot \theta_W \Theta(\rho(\lambda) - \rho) \\ g'B_i^0 &= -e|B_0^Z| (\hat{e}_Z)_i \tan \theta_W \Theta(\rho(\lambda) - \rho)\end{aligned}\tag{16}$$

We shall use two ansatze developed by Manton and Klinkhammer [3].

Ansatz (a)

$$h^{(a)}(\xi) = \begin{cases} (\xi/\Xi)^2 & \text{for } \xi < \Xi \\ 1 & \text{for } \xi > \Xi \end{cases} \quad (17)$$

$$f^{(a)}(\xi) = \begin{cases} \xi/\Omega & \text{for } \xi \leq \Omega \\ 1 & \text{for } \xi > \Omega \end{cases} \quad (18)$$

where ξ is the dimensionless variable, gFr , and Ξ and Ω are the sizes of the (dimensionless) regions outside of which the gauge field radial function, $h(\xi)$, and the higgs radial function $f(\xi)$ go to their asymptotic values respectively:

Ansatz(b)

$$h^{(b)}(\xi) = \begin{cases} \xi^2/(\Xi^2 P') & \text{for } \xi \leq \Xi \\ 1 - L(\xi) & \text{for } \xi > \Xi \end{cases} \quad (19)$$

where $P' = (1 + 4/\Xi)$ and

$$L(\xi) = \frac{4}{(4 + \Xi)} \exp \frac{1}{2}(\Xi - \xi)$$

and

$$f^{(b)}(\xi) = \begin{cases} A\xi/\Omega & \text{for } \xi \leq \Omega \\ 1 - [M(\xi)/\xi] & \text{for } \xi > \Omega \end{cases} \quad (20)$$

where

$$A = \frac{\sigma\Omega + 1}{\sigma\Omega + 2} \quad \text{and} \quad M(\xi) = \frac{\Omega}{\sigma\Omega + 2} \exp \sigma(\Omega - \xi) \quad (21)$$

and in our convention $\sigma = \lambda/g$.

We now turn to the energy of the sphaleron in the background of the Z-string (this excludes the energy of the flux tube). This corresponds to a background Z magnetic field, $B_0^Z \hat{e}_Z$.

The energy expression for the two ansatze that follows is given by the sum of the sphaleron energy in the absence of the magnetic flux, and the interaction energy, E_{int} . However, since,

the interaction energy is proportional to the field, B_0^Z , it is the modulus of E_{int} which is relevant; for its sign can always be made negative by choosing B_0^Z to be appropriately parallel or antiparallel to the positive z-axis. We then have for the energy E

$$E = E_0 - |E_{\text{int}}| \quad (22)$$

We shall give the expression for the first ansatz (a) and suppress the expression for the ansatz (b) for reasons of economy of space

$$\begin{aligned} E_0^{(a)} = \frac{4\pi F}{g} & \left[\frac{26}{35} \frac{8}{\Xi} + \left\{ \Xi \left[\frac{8}{15} - \frac{\beta}{2} + \frac{4}{15} \beta^3 - \frac{2}{35} \beta^5 \right] \Theta(\Xi - \Omega) \right. \right. \\ & \left. \left. + \Omega \left[\frac{1}{16} + \frac{16}{210} \beta^{-3} \right] \Theta(\Omega - \Xi) \right\} + \frac{\lambda^2}{g^2} \frac{\Omega^3}{105} \right] \end{aligned} \quad (23)$$

where $\beta = \Omega/\Xi$.

We shall, however, give the full expression for E_{int} for both the ansatze. For

Ansatz(a)

$$\begin{aligned} E_{\text{int}}^{(a)} = -4\pi\Gamma\Xi & \left[\left\{ \frac{16}{45} - \left(1 + \frac{1}{3} \alpha^2 \right) \gamma + \gamma^3 \left(\frac{4}{9} + \frac{\alpha^2}{3} \right) + \frac{1}{5} \gamma^5 + \frac{4}{3} \alpha^3 \arctan \left(\frac{\gamma}{\alpha} \right) \right\} \Theta(\Xi - r_0) \right. \\ & \left. + \frac{16}{45} \Theta(r_0 - \Xi) \right] \end{aligned} \quad (24)$$

where $r_0 = gv\rho_0$, $\alpha = r_0/\Xi$, $\gamma = \sqrt{1 - \alpha^2}$ and

$$\Gamma = \frac{Fe}{g^3} \lambda \cot \theta_W \quad (25)$$

Ansatz(b)

$$E_{\text{int}}^{(b)} = E_1 \Theta(\Xi - r_0) + E_2 \Theta(r_0 - \Xi) \quad (26)$$

where

$$\begin{aligned} E_1 = -4\pi\Gamma & \left\{ \left(\int_{\Xi}^{\infty} dz \int_0^{r_0^2} d(\rho^2) + \int_{\sqrt{\Xi^2 - r_0^2}}^{\Xi} dz \int_{\Xi^2 - z^2}^{r_0^2} d(\rho^2) \right) \right. \\ & \left[\frac{L(1-L)(-z^2)}{(z^2 + \rho^2)^2} + \frac{1}{4} \frac{L}{\sqrt{z^2 + \rho^2}} \frac{\rho^2}{(\rho^2 + z^2)} \right] \\ & + \frac{\Xi}{P'} \left[\frac{5}{9} + \frac{1}{3} \left(\frac{1}{P'} - 1 \right) - \frac{1}{5P'} - \left(1 + \frac{\alpha^2}{3} \right) \gamma \right. \\ & \left. \left. + \gamma^3 \left\{ \frac{4}{9} - \frac{1}{3} \left(\frac{1}{P'} - 1 \right) + \frac{\alpha^2}{3P'} \right\} + \frac{1}{P'} \frac{1}{5} \gamma^5 + \frac{4}{3} \alpha^3 \arctan \left(\frac{\gamma}{\alpha} \right) \right] \right\} \end{aligned} \quad (27)$$

and

$$E_2 = -4\pi\Gamma \left\{ \left(\int_{\Xi}^{\infty} dz \int_0^{r_0^2} d(\rho^2) + \int_0^{\Xi} dz \int_{\Xi^2 - z^2}^{r_0^2} d(\rho^2) \right. \right. \\ \left. \left. \left[\frac{L(1-L)(-z^2)}{(z^2 + \rho^2)^2} + \frac{1}{4} \frac{L}{\sqrt{z^2 + \rho^2}} \frac{\rho^2}{(\rho^2 + z^2)} \right] + \frac{\Xi}{P'} \left[\frac{5}{9} + \frac{1}{3} \left(\frac{1}{P'} - 1 \right) - \frac{1}{5P'} \right] \right) \right\} \quad (28)$$

where $r_0 = 3.35g/\lambda^{1/2}$, using the expression for $\rho_0(\lambda)$ from the previous section. We use $g^2 = 0.4$ and $\cot^2 \theta_W = 1.83$. $F = 250\text{GeV}$.

Finally, on minimizing E with respect to d and substituting this value of d in the energy we get the energy of the sphaleron in the background of the Z flux tube, E_{sp} . These are displayed in the tables. For comparison we display the usual sphaleron energies, E_{sp}^0 [3], in the absence of the flux tube, as well.

For both the ansatze, the interaction energy, E_{int} , in E_{sp} changes sign as we go from large $\lambda > 2$ to $\lambda < 2$. In our convention the sign of E_{int} for small λ is positive and that for large λ negative. Both ansatze show the presence of a minimum in E_{sp} (expected) around $\lambda \sim 0.1 - 0.2$ where E_{sp} is just slightly lower than in the absence of interaction ($\lambda = 0$). As λ increases we move to a maximum in E_{sp} around $\lambda \sim 2$, and then for larger λ , E_{sp} starts going down. For ansatz (a) (see table 1) E_{sp} is monotonically decreasing as λ increases and goes asymptotically to ~ 2 as $\lambda \rightarrow \infty$. For ansatz (b) (see table 2) E_{sp} falls much faster with increasing λ going to zero for $\lambda \sim 75$ and becoming negative for higher λ . As we have already indicated the sign of E_{int} changes around λ between 2 and 5 but since only $|E_{int}|$ occurs in the expression for E (Eq. 22), this sign is not significant.

Let us take up some of the shortcomings of this analysis which end up overestimating the sphaleron energy in the string background. The sphaleron has a size in terms of the Higgs field. This is the radial distance from the origin at which the Higgs field reaches its normal VEV. The sphaleron energy (Eq.13) is composed of three positive definite terms (leaving out the interaction term) (a) the gauge field energy, (b) the gradient energy for the Higgs field, and (c) the Higgs potential term. The term (c) is a measure of the energy due to the Higgs departing from its normal VEV. Recall that the string was constructed by minimizing the sum of the energy of emptying out the Higgs field from the flux tube and

spreading out the Z field uniformly in the crosssection of the flux tube.

If the sphaleron is contained in the flux tube then we have already accounted for the energy of emptying out the Higgs field in this region. There is no need for us to include the terms (b) and (c) above. Correcting this will reduce the energy of the sphaleron substantially.

If on the other hand the sphaleron size is larger than the flux tube radial size, then we need to add (b) and (c) above only in the region exterior to the flux tube. So we have overestimated the sphaleron energy as we included (b) and (c) inside the flux tube as well.

First, we provide a very simple illustration of the effect considered in this paper. Consider the configuration with $\Omega = \Xi = r_0$, in the parameterization of Ansatz(a). In view of the foregoing this means that the contributions of the terms (b) and (c) may be dropped in (Eq.23), as the sphaleron is inside the flux tube. The corrected sphaleron energy is

$$E_0^{(a)} = \frac{4\pi F}{g} \frac{26}{35} \frac{8}{r_0} \quad (29)$$

The interaction energy (Eq.24) is

$$E_{int}^{(a)} = -4\pi\Gamma r_0 \frac{16}{45} \quad (30)$$

and we find the ratio $\frac{E_{int}^{(a)}}{E_0^{(a)}} \sim \frac{-3}{8}$.

This shows that the sphaleron energy is already reduced by a factor 5/8 through its attractive interaction with the EW string, even for this simplified configuration which does not minimize the total energy, E .

Now, on including this correction we find that the value of λ at which the $E_{sp} \leq 0$, (Ansatz(b)), comes down to $\lambda \sim 60$ as compared to $\lambda \sim 80$, in the absence of the correction. This is already a substantial lowering. We do not exhibit this correction for the whole table but just quote the new value of $\lambda \sim 60$ at which $E_{sp} \leq 0$.

We have used the parametrization [3] to calculate the sphaleron energy. Also, the energy, radius and Z magnetic field for the Z-string are estimated as in Ref. [6] without using the exact solutions in [4] and [5]. To get reliable numbers an exact solution is required. In this

case we expect that E_{sp} will come down substantially and go to zero for smaller λ than those quoted above.

CONCLUSIONS

We have found that in the background of the EW Z-string, E_{sp} , that is, the instanton potential barrier relevant to B violation is reduced by interaction. In terms of catalysing B-violation, however, the reduction of the barrier becomes significant only for rather large $\lambda \sim 60$, when the barrier goes to 0. This value of λ is unrealistic for the standard EW model where we expect $\lambda \sim 1$.

When $E_{sp} < 0$ we have a qualitatively new situation. The sphaleron no longer has the interpretation of an instanton potential barrier. It now has an energy that is lower than the winding number (PG) vacua whose energy is not affected by the flux tube. Such a sphaleron can bind to the flux tube/string and lower its energy; one thus expects that it will be spontaneously produced on the flux tube. This happens for $\lambda \geq 60$. In this regime, $E_{sp} < 0$, the ground state string solution will have sphaleron bound states beading on it. This is a new string solution.

Furthermore, sphaleron bound states would endow the EW Z-strings with winding number (N_{cs}) and consequently baryon number. If such strings were created during the EWPT they would decay to the vacuum well after the EWPT giving rise to B violation and a B asymmetry in the presence of CP violation. This could yield some of the effects considered in [5] without any looping and twisting.

It is intriguing that if we had a flux quantum an order of magnitude larger (than it actually is) for the EW string we would have found $E_{sp} = 0$ for $\lambda \sim 1$! Therefore it is important to consider an exact string solution where the magnetic field is much higher at the origin and falling exponentially out in contrast to the averaged out field we have used in Sec.2. Also, an exact numerical solution for the sphaleron instead of the ansatze we have used is important. It is not clear how sensitive our results are to these details. It is evident

that such a program must be importantly and urgently carried out to get definitive answers.

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TABLES

λ	E_{sp}^0	Ξ^0	Ω^0	E_{sp}	E_{int}	Ξ	Ω
0.001	2.40	4.95	4.8	2.399	0.002	4.95	4.8
0.1	2.43	4.81	4.53	2.18	0.262	5.31	4.98
0.2	2.48	4.56	4.05	2.015	0.469	4.77	4.209
1.0	2.92	3.61	1.99	2.26	0.92	2.48	1.66
2.0	3.17	3.41	1.19	2.62	1.105	1.92	1.02
5.0	3.38	3.34	0.51	3.08	− 0.33	3.69	0.52
10.0	3.47	3.34	0.263	2.94	− 0.535	3.4	0.26
100.0	3.55	3.34	0.0264	2.46	− 1.187	2.85	0.026
10000.0	3.56	3.33	0.00053	2.03	− 2.035	1.97	0.00026

TABLE I.

λ	E_{sp}^0	Ξ^0	Ω^0	E_{sp}	E_{int}	Ξ	Ω
0	1.566	2.66	2.60	1.566	0	2.66	2.60
0.1	1.67	2.15	2.37	1.508	0.154	2.5	2.59
0.2	1.77	1.81	2.05	1.54	0.47	2.25	2.30
0.5	1.95	1.39	1.60	1.709	0.244	1.5	1.65
1.0	2.13	1.10	1.19	1.928	0.174	1.2	1.22
2.0	2.32	0.892	0.795	2.309	0.009	0.9	0.8
10.0	2.61	0.733	0.21	1.827	− 0.945	0.225	0.198
80.0	2.708	0.729	0.026	− 0.06	− 3.343	0.007	0.026
100.0	2.711	0.729	0.021	− 0.324	− 3.634	0.0042	0.021

TABLE II.

Table Caption(s)

Table I. Ansatz(a): Parameters for the sphaleron interaction with the Z string/flux tube. The superscript ⁰ indicates the values of the variables for the case of no interaction which match with those of Ref. [3]. The units of energy are $(4\pi F v/g)$ again conforming to the convention used in Ref. [3]. The interaction energy changes sign around $\lambda \sim 2$, though it is only the modulus of E_{int} which is relevant for E_{sp}

Table II. Ansatz(b): Table caption same as for Table I.